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A Matlab Tool for Frequency-Domain Identification of Radiation-Force Models of Ships and Offshore Structures

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Contents

Replacements

This is a new technical report.

Executive summary

This article describes a Matlab tool for parametric identification of radiation-force models of marine structures. These models are a key component of force-to-motion models used in simulators, motion control designs, and also for initial performance evaluation of wave-energy converters. The software described provides tools for preparing the non-parmatric data generated hydrodynamic codes and identification with automatic model-order detection. The identification is considered in the frequency domain.

1 Introduction

One approach to develop linear time-domain models of marine structures consist of using potential-theory hydrodynamic codes to compute frequency-dependent coefficients and frequency responses, and then use these data for system identification in order to implement the Cummins equation, which is a linearised vector equation of motion. If physical-model or full scale experiments are available, then mmathematical model based on the Cummins equation can be corrected for viscous effects. This procedure us summarised in Figure 1.

A great deal of work has been reported in the literature proposing the use of different identification methods to obtain approximating fuild-memory models. Taghipour et al. (2008) and Perez and Fossen (2008b) provide an up-to-date review of the different methods. In particular, the latter reference discusses the advantages of using frequency-domain methods for the identification of fluid memory models. Since the data provided by hydrodynamic codes is in the frequency domain, identification in the frequency domain is a natural approach, which does not require transformation of the data to the time domain. If not handled appropriately, the latter transformation can result in errors due to the finite amount of frequency-domain data. More importantly, when performing frequency-domain identification, one can enforce model structure and parameter constraints; and thus, the class of models over which the search is done is reduced, and the models obtained present characteristics in agreement with the hydrodynamic modelling hypothesis.

In this article, we present a set of Matlab functions to perform identification of radiation forces. We consider two cases. In first case, information related to the infinite-frequency added mass coefficients is considered available. In the second case, these coefficients are estimated jointly with the fluid-memory model (Perez and Fossen, 2008a). The second case is relevant for hydrodynamic codes based on 2D-potential theory, which do not normally solve the boundary-value problem associated with the innite frequency.

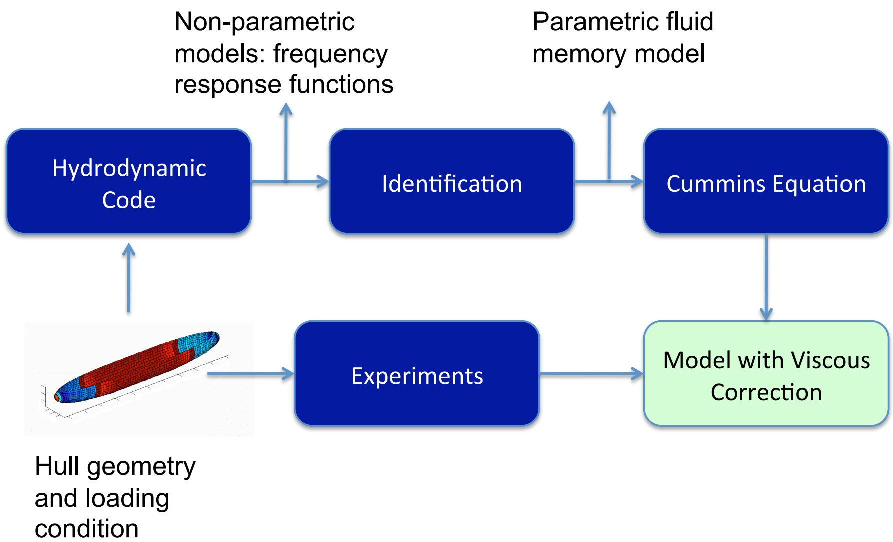

Figure 1: Hydrodynamic modelling procedure.

2 Dynamics of Ships and Offshore Strucutres

The linearised equation of motion of marine structure can be formulated as

$$\mathbf{M}_{RB}\,\ddot{\boldsymbol{\xi}} = \boldsymbol{\tau}. \tag{1}$$

The matrix $\mathbf{M}_{RB}$ is the rigid-body generalised mass. The generalised-displacement vector $\boldsymbol{\xi} \triangleq [x, y, z, \phi, \theta, \psi]^T$ gives the position of the body-fixed frame with respect to an equilibrium frame (x-surge, y-sway, and z-heave) and the orientation in terms of Euler angles (ϕ-roll, θ-pitch, and ψ-yaw). The generalised force vector and $\boldsymbol{\tau} \triangleq [X, Y, Z, K, M, N]^T$ gives the respective forces and moments in the six degrees of freedom. This force vector can be separated into three components:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{rad} + \boldsymbol{\tau}_{visc} + \boldsymbol{\tau}_{res} + \boldsymbol{\tau}_{exc}, \tag{2}$$

where the first term corresponds to the radiation forces arising from the change in momentum of the fluid due to the motion of the structure and the waves generated as the result of this motion, the second term corresponds to forces due to fluid viscous effects, the third term corresponds to restoring forces due to gravity and buoyancy, and the fourth component represents the pressure forces due to the incoming waves other forces used to control the motion of the marine structure.

Cummins (1962) used potential theory to study the radiation hydrodynamic problem in the time-domain for an ideal fluid (no viscous effects) and found the following representation:

$$\boldsymbol{\tau}_{rad} = -\mathbf{A}_{\infty}\,\ddot{\boldsymbol{\xi}} - \int_0^t \mathbf{K}(t-t')\dot{\boldsymbol{\xi}}(t')\,dt'. \tag{3}$$

The first term in (3) represents pressure forces due the accelerations of the structure, and $\mathbf{A}_{\infty}$ is a constant positive-definite matrix called *infinite-frequency added mass.* The second term represents fluid-memory effects that capture the energy transfer from the motion of the structure to the radiated waves. The convolution term is known as a *fluid-memory model.* The kernel of the convolution term, $\mathbf{K}(t)$, is the matrix of *retardation* or *memory functions* (impulse responses).

By combining terms and adding the linearised restoring forces $\boldsymbol{\tau}_{res} = -\mathbf{G}\boldsymbol{\xi}$, the *Cummins Equation* (Cummins, 1962) is obtained:

$$(\mathbf{M}_{RB} + \mathbf{A}_{\infty})\ddot{\boldsymbol{\xi}} + \int_0^t \mathbf{K}(t-t')\dot{\boldsymbol{\xi}}(t')\,dt' + \mathbf{G}\boldsymbol{\xi} = \boldsymbol{\tau}_{exc}, \tag{4}$$

Equation (4) describes the motion of ships and offshore structures in an ideal fluid provided the linearity assumption is satisfied. This model can then be embellished with non-linear components taking into account, for example, viscous effects and mooring lines–see Figure 1.

3 Frequency-domain Models

When the radiation forces (3) are considered in the frequency domain, they can be expressed as follows (Newman, 1977; Faltinsen, 1990):

$$\boldsymbol{\tau}_{rad}(j\omega) = -\mathbf{A}(\omega)\ddot{\boldsymbol{\xi}}(j\omega) - \mathbf{B}(\omega)\dot{\boldsymbol{\xi}}(j\omega). \tag{5}$$

The parameters $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ are the *frequency-dependent added mass* and *potential damping* respectively. This representation leads to the following frequency-domain relationship between the excitation forces and the displacements:

$$[-\omega^2[\mathbf{M} + \mathbf{A}(\omega)] + j\omega\mathbf{B}(\omega) + \mathbf{G}]\boldsymbol{\xi}(j\omega) = \boldsymbol{\tau}_{exc}(j\omega). \tag{6}$$

Ogilvie (1964) showed the realation between the parameters of the time-domain model (4) and frequency-domain model (6) using the Fourier Transform of (4):

$$\mathbf{A}(\omega) = \mathbf{A}_{\infty} - \frac{1}{\omega}\int_0^{\infty} \mathbf{K}(t)\,\sin(\omega t)\,dt, \tag{7}$$

$$\mathbf{B}(\omega) = \int_0^{\infty} \mathbf{K}(t)\,\cos(\omega t)\,dt. \tag{8}$$

From expression (7) and the application of the Riemann-Lebesgue lemma, it follows that $\mathbf{A}_{\infty} = \lim_{\omega\to\infty} \mathbf{A}(\omega)$, and hence $\mathbf{A}_{\infty}$ is called infinite-frequency added mass.

It also follows from the Fourier transform that the time- and frequency-domain representation of the retardation functions are

$$\mathbf{K}(t) = \frac{2}{\pi}\int_0^{\infty} \mathbf{B}(\omega)\,\cos(\omega t)\,d\omega, \tag{9}$$

and

$$\mathbf{K}(j\omega) = \mathbf{B}(\omega) + j\omega[\mathbf{A}(\omega) - \mathbf{A}_{\infty}]. \tag{10}$$

Expressions (9) and (10) are key to generate the data used in the identification problems that seek parametric approximations to the fluid memory terms.

Hydrodynamic codes based on potential theory, are nowadays readily to compute $\mathbf{B}(\omega)$ and $\mathbf{A}(\omega)$ for a finite set of frequencies of interest—see, for example, Beck and Reed (2001) and Bertram (2004) for an overview. Hydrodynamic codes based on 3D potential theory usually solve, in addition, the boundary-value problem associated with infinite-frequency that gives $\mathbf{A}_{\infty}$.

4 Identification of Radiation-force Models

A direct approach to use non-parametric models to implement simulation models consists of a direct implementation of (4) in discrete time. This approach can be time consuming and may require significant amounts of computer memory. In addition, the non-parmteric models can result difficult to work with for the analysis and design of vessel motion control systems.

One way to overcome these difficulties consists of approximating the fluid-memory models by a linear-time-invariant parametric model in state-space form:

$$\boldsymbol{\mu} = \int_0^t \mathbf{K}(t-t')\dot{\boldsymbol{\xi}}(t')\,dt' \quad \approx \quad \begin{array}{l} \dot{\mathbf{x}} = \hat{\mathbf{A}}\,\mathbf{x} + \hat{\mathbf{B}}\,\dot{\boldsymbol{\xi}} \\ \hat{\boldsymbol{\mu}} = \hat{\mathbf{C}}\,\mathbf{x}, \end{array} \tag{11}$$

where the number of components of the state vector $\mathbf{x}$ corresponds to the order of the approximating system and the matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, and $\hat{\mathbf{C}}$ are constants. Note that the above state-space approximation does not have a feed-throug term $\hat{\mathbf{D}}\,\dot{\boldsymbol{\xi}}$ in the output equation. The reason for this is that the mapping $\dot{\boldsymbol{\xi}} \mapsto \boldsymbol{\mu}$ has relative degree 1—this is discussed in the next section.

The above approximation problem can be considered in the frequency domain:

$$\mathbf{K}(j\omega) \approx \hat{\mathbf{K}}(s = j\omega), \tag{12}$$

where $\hat{\mathbf{K}}(s)$ is matrix of rational transfer functions:

$$\hat{K}_{ik}(s) = \frac{P_{ik}(s)}{Q_{ik}(s)} = \frac{p_r s^r + p_{r-1} s^{r-1} + ... + p_0}{s^n + q_{n-1} s^{n-1} + ... + q_0}. \tag{13}$$

4.1 Identification when A_∞ is Avalaible

The identification problem then consists of selecting the order of the transfer functions $\hat{K}_{ik}(s)$ (13) and estimating their parameters. This problem can be formulated in terms of curve fitting:

$$\boldsymbol{\theta}^\star = \arg\min_{\boldsymbol{\theta}} \sum_l w_l\, \epsilon_l^*\, \epsilon_l, \tag{14}$$

$$\epsilon_l = K_{ik}(j\omega_l) - \hat{K}_{ik}(j\omega_l, \boldsymbol{\theta}) \tag{15}$$

where the notation * indicates transpose complex conjugate, w_l are weighting coefficients. The non-paramtetric model $K_{ik}(j\omega_l)$ is computed via (10) using $A_{ik}(\omega_l)$, $B_{ik}(\omega_l)$ and $A_{\infty,ik}$, which are obtained from a hydrodynamic code for a set of frequencies ω_l. The structure of the estimate $\hat{K}_{ik}$ is given by (13), and the vector of parameters $\boldsymbol{\theta}$ is given by

$$\boldsymbol{\theta} = [p_r, ..., p_0, q_{n-1}, ..., q_0]^T. \tag{16}$$

From hydrodynamic properties of the model, it follows that the problem (14) must be considered subject to the following constraints (Perez and Fossen, 2008b):

$$\hat{K}_{ik}(s)\,\text{has a zero at}\, s = 0, \tag{17}$$

$$\hat{K}_{ik}(s)\,\text{has relative degree 1}, \tag{18}$$

$$\hat{K}_{ik}(s)\,\text{is stable}, \tag{19}$$

$$\hat{K}_{ik}(s)\,\text{is positivie real for}\, i = k. \tag{20}$$

The above is a non-linear optimisation problem. Two methods can be followed to solve this problem:

1. Linearise (14), and solve a sequence of linear Least-Square problems using the solution of the previous iteration to compute the weighting coefficients w_l.

2. Use the solution of the linear problem to initialise a Gauss-Newton search algorithm.

The linearisation of (14) is due to (Levy, 1959) and the iterative solution via a sequence of linear problems is due to (Sanathanan and Koerner, 1963):

$$\hat{\boldsymbol{\theta}}_p = \arg\min_{\boldsymbol{\theta}} \sum_l s_{l,p} \left| Q_{ik}(j\omega_l, \boldsymbol{\theta}) K_{ik}(j\omega_l) - P_{ik}(j\omega_l, \boldsymbol{\theta}) \right|^2, \tag{21}$$

where

$$s_{l,p} = \frac{1}{|Q_{ik}(j\omega_l, \hat{\boldsymbol{\theta}}_{p-1})|^2}.$$

Note that (21) results in a Linear LS minimization. After a few iterations (usually p=10 to 20), $Q_{ik}(j\omega_l, \boldsymbol{\theta}_p) \approx Q_{ik}(j\omega_l, \boldsymbol{\theta}_{p-1})$; and therefore, (14) is approximately recovered. This allows solving the nonlinear LS problem via an iteration of linear ones.

This method provides a good trade-off between accuracy and computational complexity. The toolbox described in Section 5 supports both methods.

4.2 Order Selection

With respect to the order selection of the approximation, it follows from the constraints (17)-(20) that the minimum order transfer function has the following form

$$\hat{K}_{ik}^{\min}(s) = \frac{p_1 s}{s^2 + q_1 s + q_0}. \tag{22}$$

For automatic order determination, one can start with the lowest order approximation and increase the order to improve the fitting until a satisfactory approximation is obtained. This is the method used in the software described in Section 5. As a metric for determining the quality of the fit the coefficient of determination $0 \leq R^2 \leq 1$ is used for both added mass and damping:

$$R^2 = 1 = \frac{\sum_k (X_k - \hat{X}_k)^2}{\sum_k (X_k - \overline{X})^2}, \tag{23}$$

where X_k are the data points and $\hat{X}_k$ are the estimates. That is once the parametric model (13) is obtained, the added mass and damping are reconstructed from its real and imaginary parts, and the corresponding coefficients of determination are computed. If these coefficients are below 0.99, then the model order is increased.

4.3 Stability

The resulting model from the LS minimization may not necessarily be stable because stability is not enforced as a constraint in the optimisation. This can be addressed after the identification. Should the obtained model be unstable, one could obtain a stable one by reflecting the unstable poles about the imaginary axis and re-computing the denominator polynomial. That is,

(i) Compute the roots of $\lambda_1, \ldots, \lambda_n$ of $Q_{ik}(s, \hat{\boldsymbol{\theta}}_{ik})$.

(ii) If $\text{Re}\{\lambda_i\} > 0$, then set $\text{Re}\{\lambda_i\}$ = - $\text{Re}\{\lambda_i\}$,

(iii) Reconstruct the polynomial:
$Q_{ik}(s) = (s - \lambda_1)(s - \lambda_1) \cdots (s - \lambda_n)$.

4.4 Passivity

The mapping $\dot{\boldsymbol{\xi}}$ into a force introduced by the fluid-memory convolution is passive–see Perez and Fossen (2008a). A disadvantage of the method of LS curve fitting is that it does not enforce passivity. If passivity is required (i.e., $B_{ik}(\omega) > 0$), a simple way to ensure it is to try different order approximations and choose the one that is passive. The approximation is passive if

$$\mathrm{Re}\left\{\frac{P_{ik}(j\omega_l, \boldsymbol{\theta})}{Q_{ik}(j\omega_l, \boldsymbol{\theta})}\right\} > 0. \tag{24}$$

When this is checked, one should evaluate the transfer function at low and high frequencies—below and above the frequencies used for the parameter estimation.

Often, low-order approximations models of the convolution terms given by this method are passive—the term 'low' depends on the data of the particular vessel under consideration. Therefore, one can reduce the order and trade-off fitting accuracy for passivity.

4.5 Identification when $\mathbf{A}_\infty$ is not Availaible

Hydrodynamic codes based on 2-D potential theory normally do not provide the value of the infinite-frequency added mass matrix $\mathbf{A}_\infty$. In these cases, we cannot form $\mathbf{K}(j\omega)$ as indicated in (10).

In this section, we summarise the method of Perez and Fossen (2008a), which estimates jointly the infinite-frequency added mass and the fluid-memory transfer function. The method exploits the knowledge and procedures used in the identification of $\hat{K}_{ik}(s)$ discussed in the previous section, and therefore, it provides an extension of those results putting the two identification problems into the same framework.

On the one hand, the radiation forces in the frequency-domain given in (5) can be expressed

$$\tau_{rad,i}(j\omega) = -\left[\frac{B_{ik}(\omega)}{j\omega} + A_{ik}(\omega)\right]\ddot{\xi}_k(s), \tag{25}$$

where the expression in brackets gives the complex coefficient

$$A_{ik}(j\omega) \triangleq \frac{B_{ik}(\omega)}{j\omega} + A_{ik}(\omega). \tag{26}$$

On the other hand, taking the Laplace transform of (3), and assuming a rational approximation for the convolution term we obtain

$$\hat{\tau}_{rad,i}(s) \;\; = -\left[A_{\infty,ik}\, s + \tfrac{P_{ik}(s)}{Q_{ik}(s)}\right]\dot{\xi}_k(s), \tag{27}$$

$$= -\left[A_{\infty,ik} + \tfrac{P'_{ik}(s)}{Q_{ik}(s)}\right]\ddot{\xi}_k(s) \tag{28}$$

The transfer function in brackets in (28) can be further expressed as

$$\hat{A}_{ik}(s) = \frac{R_{ik}(s)}{S_{ik}(s)} = \frac{A_{\infty,ik}Q_{ik}(s) + P'_{ik}(s)}{Q_{ik}(s)}. \tag{29}$$

Thus, we can use Least-Squares optimisation to estimate the parameters of the approximation (29) given the frequency-respose data (26):

$$\boldsymbol{\theta}^\star = \arg\min_{\boldsymbol{\theta}} \sum_l w_l\,(\epsilon_l^*\,\epsilon_l), \tag{30}$$

with

$$\epsilon_l = A_{ik}(j\omega_l) - \frac{R_{ik}(j\omega_l, \boldsymbol{\theta})}{Q_{ik}(j\omega_l, \boldsymbol{\theta})}, \tag{31}$$

and the constraint that $n = \deg R_{ik}(s) = \deg R_{ik}(s)$. As already mentioned in the previous section, the minimum order approximation is $n = 2$. Therefore, we can start with this order and increase it to improve the fit if necessary. Hence, we can use the same algorithms that we use for the case when the infinite-frequency added mass is available, subject to different order constraints in interpretation of the estimates obtained.

Since the polynomial $Q_{ik}(s)$ is normalised to be monic, then

$$\hat{A}_{\infty,ik} = \lim_{\omega\to\infty} \frac{R_{ik}(s, \boldsymbol{\theta}^\star)}{S_{ik}(s, \boldsymbol{\theta}^\star)}. \tag{32}$$

That is, the infinite-frequency added mass $A_{\infty,ik}$ is the coefficient of the highest order term of $R_{ik}(s, \boldsymbol{\theta}^\star)$. Also, after obtaining $R_{ik}(s, \boldsymbol{\theta}^\star)$ and $S_{ik}(s, \boldsymbol{\theta}^\star)$, we can recover the polynomials for the fluid-memory model:

$$\begin{aligned} Q_{ik}(s, \boldsymbol{\theta}^\star) &= S_{ik}(s, \boldsymbol{\theta}^\star), \\ P_{ik}(s, \boldsymbol{\theta}^\star) &= R_{ik}(s, \boldsymbol{\theta}^\star) - \hat{A}_{\infty,ik} S_{ik}(s, \boldsymbol{\theta}^\star). \end{aligned} \tag{33}$$

5 Toolbox Description

The toolbox presented in this paper is an independent component of the Marine Systems Simulator (MSS, 2009). Figure 2 shows a diagram of the different software components of the toolbox, and their dependability. The main function of the toolbox is `FDIRadMod.m`, which processes the input data and returns the estimate of the fluid memory transfer function and also the infinite-frequency added mass if required. This function calls other functions to prepare the data for identification and to compute the estimates. The toolbox also includes two demos which show how to use the main function. The first demo considers the estimation with infinite-frequency added mass available (WA), and the second demo considers the estimation when infinite-frequency added mass is not available (NA).

The functionality of the main components is described in the following.

5.1 FDIRadMod.m

Purpose: This function process the input data and generates a transfer function object with the estimate fluid-memory model. The function processes only a single-input-single-output models. Hence, for a multiple degree of freedom structure, this function should be used for each relevant coupling i, k.

Syntax:

```
[Krad,Ainf_hat]=FDIRadMod(W,A,Ainf,B,FDIopt,Dof)
```

Input Data:

- `W` – Vector of frequencies.
- `A` – Vector of frequency-dependant added mass.
- `Ainf` – Infinite-frequency added mass.

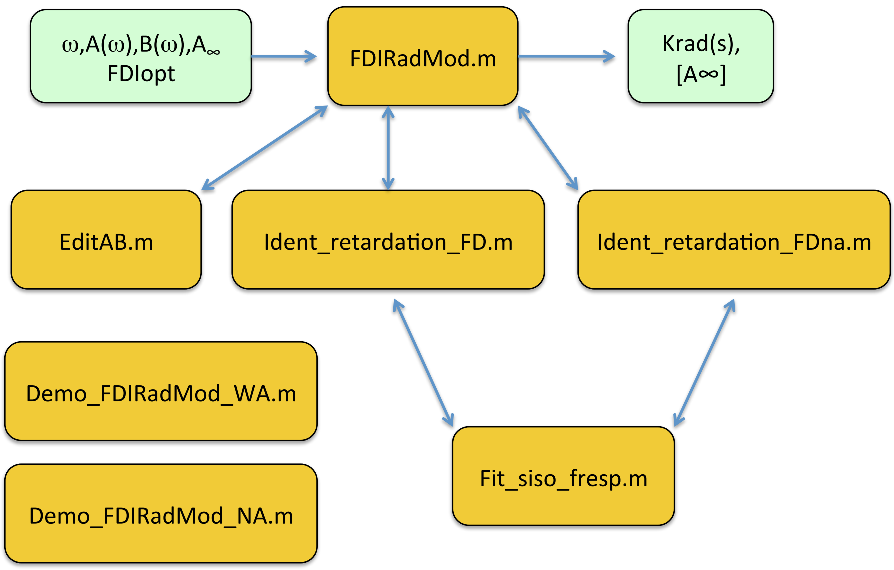

Figure 2: Software organisation and dependability.

- `B` – Vector of frequency-dependant potential damping.
- `FDIopt` – Structure with computation options.
- `Dof = [i,k]` – Coupling of degrees of freedom.

The structure `FDIopt` has the following fields

- `FDIopt.OrdMax` – Maximum order to be used in automatic order detection. Typical value 20.
- `FDIopt.AinfFlag` – Logic flag. If set to 1, the value `Ainf` is used in the calculations. If set to 0, the infinite- frequency added mass is estimated and the value in the argument of the function is ignored.
- `FDIopt.Method` – This refers to the methods used to solve the parameter optimisation problem. Option 1 uses a a linearised model and linear Least Squares. Option 2 uses iterative linear Least Squares, Option 3 uses the linear Least Squares solution to initialise a non-linear Least-Square problem solved using the Gauss-Newton method. The option value 2 gives a good trade-off between computational speed and accuracy.
- `FDIopt.Iterations` – Maximum number of iterations to be used in the iterative linear LS solution.
- `FDIopt.LogLin` – Logic flag. If set to 1 all the data is plotted in logarithmic scale. If set to 0, all the data is plotted in linear scale.
- `FDIopt.wsFactor` – This is a sampling factor for plotting the data of the parametric approximation. The sample frequency used to plot the data is this factor times the minimum difference of frequencies in the input vector `W`. A typical value is 0.1.
- `FDIopt.wminFactor` – The minimum frequency to be used in the plot is `FDIopt.wminFactor*Wmin`, where `Wmin` is the minimum frequency of the dataset used for identification. Typical value 0.1.

- `FDIopt.wmaxFactor` – The maximum frequency to be used in the plot is `FDIopt.wmaxFactor*Wmax`, where `Wmax` is the maximum frequency of the dataset used for identification. Typical value 2 to 5.

Output Data:

- `Krad` – Single-input-single-output transfer function object with the estimate of the fluid memory approximation.
- `Ainf_hat` – Estimate of the infinite-frequency added mass coeffieicient. If the option `FDIopt.AinfFlag` is set to 1, then `Ainf_hat=Ainf`.

Description: The function `FDIRadMod.m` first calls `editAB.m` to prepare the data for identification, which allows the used to select the frequency range to be used for identification and to eliminate wild points that may be present in the data due to numerical problems associated with the hydrodynamic code. Then, depending on `FDIopt.AinfFlag`, the function calls the appropriate computation routine—see Figure 2.

The function `FDIRadMod.m` also makes an automatic order estimate by increasing the order of the approximation and computing the the coefficient of determination related to the fitting of both added mass and damping. When this coefficient reaches the value 0.99, the function stops increasing the order, and the re-constructued added mass and damping are plotted together with the nonparametric data used for identification. At this point, the function prompt the user to either adjust the order of the approximation manually via a keyboard input or either to leave the model as it is and exit the function. The user can make as many changes in order as required, and every time there is a change in the order, the model is re-estimted.

5.2 EditAB.m

Purpose: This function allows the user to select the frequency ranged to be used for identification and to eliminate data wild points[1]. This is a support function for `FDIRadMod.m`, so the used may not need to call it directly.

Description: The function first plots the added mass and potential damping as a function of the frequency, and then prompts the user to select the range of freuencies to for idnetification. This is done by clicking with the mouse on the plot of either the added mass and damping. The low-frequency data point should be clicked first, followed by the high-frequency point, and finally press return. The selected data is then re-plotted.

After selecting the frequency range, the function allows the elimination of data wild points. A message on the workspace prompts the user to opt for wild point elimination. If required, this elimination is done by clicking with the mouse on all the points that are to be eliminated, and then press enter—this can be done either on the added mass or potential damping plot. The function allows the user to re-start the process in case a point is deleted accidentally.

[1] Wild points in the data of hydrodynamic codes are due to ill-conditioned numerical problems, which normally arise at high-frequencies because of inappropriate panel sizes used to discreteise the hull–see Faltinsen (1990) for details.

5.3 Ident_retardation_FD.m

Purpose: This function performs the parameter estimation of the approximating fluid-memory transfer function given a desired order and the frequency response $K(j\omega_l)$.

Description: This is a support function for `FDIRadMod.m`, so the user may not need to call it directly. This function performs the estimation for the problem in which the infinite frequency added mass is available to compute $K(j\omega_l)$. This problem is described in Section 4.1. The function performs data scaling and enforces the model structure constraints (17)–(20). A summary of the algorithm main steps is given in the Appendix.

5.4 Ident_retardation_FDna.m

Purpose: This function performs the joint parameter estimation of the approximating fluid-memory transfer function and the infinite-frequency added mass coefficient.

Description: This is a support function for `FDIRadMod.m`, so the user may not need to call it directly. The function uses as input the frequency-dependant added mass and damping, and it requires a desired order. This problem is described in Section 4.5. The function performs data scaling and enforces the model structure constraints (17)–(20).

5.5 Fit_siso_fresp.m

Purpose: This is a general purpose function to estimate single-input-single-output transfer function of a specified order and relative degree given a complex frequency response.

This function can not only be used to identify fluid-memory transfer function, but also force to identify force-to-motion transfer functions (Perez and Lande, 2006). That latter is a functionality that will be included in future versions of the toolbox.

Description: This is a support function for `Ident_retardation_FD[na].m`, so the user may not need to call it directly. This function implements 3 methods for parameter estimation, namely, 1 - uses a a linearised model and linear Least Squares. 2 - uses iterative linear Least Squares, 3- uses the linear Least Square solution to initialise a non-linear Least Square problem solved using the Gauss-Newton method. The function is build upon the functionality `invfreqs.m` of Matlab's Signal Processing Toolbox.

If the user does not have access to `invfreqs.m`, then the algorithm given in the Appendix could be easily implemented by the user—this algorithm refers to the method 2 above, namely, iterative linear Least Squares.

6 Demos

The toolbox provides two demo files than make use of the main function `FDIRadMod.m`–see Figure 2. These demos are based on the data of a FPSO that belongs to the Hydro-add in of the Marine Systems Simulator (MSS, 2009).

The first demo, `Demo_FDIRadMod_WA.m` (WA-with infinite-frequency Added mass), loads the `vessel` data structure, and allows the user to select the desired coupling (i, k) for identification. The structure `vessel` contains data corresponding to 6 degrees of freedom. Hence i, k =1,...,6. In this section, we will illustrate the results on the models corresponding to vertical motion; that is, couplings 3-3, 3-5, 5-3, and 5-5.

Figure 3 shows the raw added mass and damping for coupling 5-3, which by symmetry of the hull it is the same as the 3-5 coupling. This is the data obtained from the hydrodynamic code. Figure 4 shows the edited data after eliminating some wild points. Figure 5 shows the corresponding curve fitting results. This figure shows the fitting of the fluid-memory frequency response on the left-hand column, and the re-construction of added mass and damping on the right-hand column. The order of the approximation is 5, which is obtained automatically by the function. Figures 6 and 7 show the corresponding results for the 3-3 and 5-5 couplings. For both these couplings the automatic order detection selected order 3, but then we manually increase the order to 4 to have a better fit.

The second demo, `Demo_FDIRadMod_NA.m` (NA-No infinite-frequency Added mass), also loads the `vessel` data structure of the FPSO, and allows the user to select the desired coupling for identification. In this demo, however, the identification is done without using the infinite-frequency added mass coefficient. Figure 8 shows the fitting results for the 5-3 coupling. The left-hand column shows the fittign of the complex coefficient $\tilde{A}(j\omega)$ given by (26), whereas the right-hand column shows the re-construction of added mass and damping. Figure 9 shows the estimated fluid-memory frequency response function. These results are in agreement with those shown in Figure 5; however, there are small differences due to the fact that the two estimators use different information.

It is worthwhile highlighting that the removal of wild point is important. For example, Figures 10 and 11 shows the results of identification without using added mass for the coupling 5-3 when the wild points in the added mass and damping have not been removed. In this case the automatic order detection selected an approximation of order 10. This is because the algorithm tries to fit highly resonant poles to the wild points. In these cases the function gives the user the option to manually reduce the order. However, for some cases this may not solve the problem, and the identification process should be started again and remove the wild points.

7 Software Repository

The toolbox presented in this report is an independent component of the Marine Systems Simulator (MSS, 2009) maintained by the authors. This is a free toolbox released under a GNU licence. The software is under continuing development and it is available at `www.marinecontrol.org`

8 Conclusion

This paper describes a toolbox for identification of radiation force models of marine structures using frequency-domain identification. The models identified find application in the development of ship simulators, control design, and the evaluation of wave energy converters.

The software described provides tools for preparing the non-parmatric data generated hydrodynamic codes, automatic model order detection, and parameter estimation. The toolbox contains a main function that performs all these tasks by calling other support functions. The user may only need to call the main function.

The identification is done for single-input-single-output models. This give freedom to the user to select the couplings of interest each particular application and to integrate the functionality of the toolbox into other data processing codes.

The toolbox uses the function `invfreqs.m` of Matlab's Signal Processing Toolbox. If the user does not have access to `invfreqs.m`, then the algorithm given in the Appendix could be easily implemented by the user—this algorithm refers to the iterative linear Least Square solution of the parameter estimation problem.

The software is part of the Marine Systems Simulator and is available at `www.marinecontrol.org`

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A Parameter Estimation Algorithm

This section presents the main steps of the parameter estimation algorithm for the case where the infinite-frequency added mass coefficients are used in the identification.

1. Set the appropriate range of frequencies where the hydrodynamic data is considered accurate, eliminate wild points, and compute the frequency response for a set of frequencies ω_l:
$$K_{ik}(j\omega_l) = B_{ik}(\omega_l) - j\omega(A_{ik}(\omega_l) - A_{\infty,ik}). \tag{34}$$

2. Scale the data:
$$K'_{ik}(j\omega_l) = \alpha\, K_{ik}(j\omega_l), \qquad \alpha \triangleq \frac{1}{\max|K_{ik}(j\omega_l)|}. \tag{35}$$

3. Select the order of the approximation n=deg($Q_{ik}(j\omega, \boldsymbol{\theta}_{ik})$). The minimum order approximation n=2 can be the starting point and can be used for automatic order selection.

4. Estimate the parameters
$$\boldsymbol{\theta}^{\star}_{ik} = \arg\min_{\boldsymbol{\theta}} \sum_l \left| \frac{K'_{ik}(j\omega_l)}{(j\omega_l)} - \frac{P'_{ik}(j\omega_l, \boldsymbol{\theta})}{Q_{ik}(j\omega_l, \boldsymbol{\theta})} \right|^2, \tag{36}$$
with deg($P'_{ik}(j\omega, \boldsymbol{\theta}_{ik})$)=$n-2$. This problem can be linearised and solved iteratively as in (21).

5. Check stability by computing the roots of $Q_{ik}(j\omega, \boldsymbol{\theta}^{\star}_{ik})$ and change the real part of those roots with positive real part.

6. Construct the desired transfer function by scaling and incorporate the s factor in the numerator:
$$\hat{K}_{ik}(s) = \frac{1}{\alpha}\,\frac{s\,P'_{ik}(s, \boldsymbol{\theta}^{\star}_{ik})}{Q_{ik}(s, \boldsymbol{\theta}^{\star}_{ik})}. \tag{37}$$

7. Estimate the added-mass and damping based on the identified parametric approximation via
$$\hat{A}_{ik}(\omega) = \mathrm{Im}\{\omega^{-1}\,\hat{K}_{ik}(j\omega)\} + A_{\infty,ik} \tag{38}$$
$$\hat{B}_{ik}(\omega) = \mathrm{Re}\{\hat{K}_{ik}(j\omega)\}, \tag{39}$$
and compare with the $A_{ik}(\omega)$ and $B_{ik}(\omega)$ given by the hydrodynamic code. If the fitting is not satisfactory increase the order of the approximation and go back to step (iii).

8. Check for passivity if required $\hat{B}_{ik}(j\omega) > 0$.

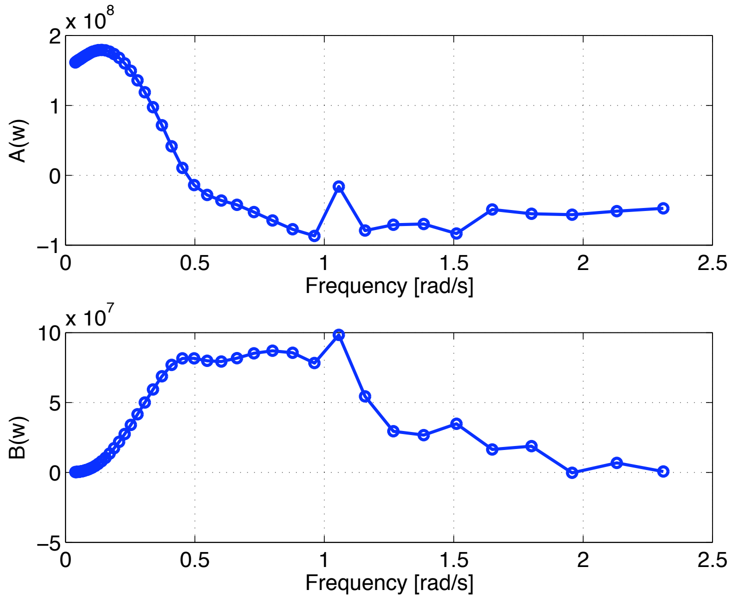

Figure 3: Raw added mass and damping data of a FPSO vessel computed by a hydrodynamic code. Coupling 5-3 (pitch-heave).

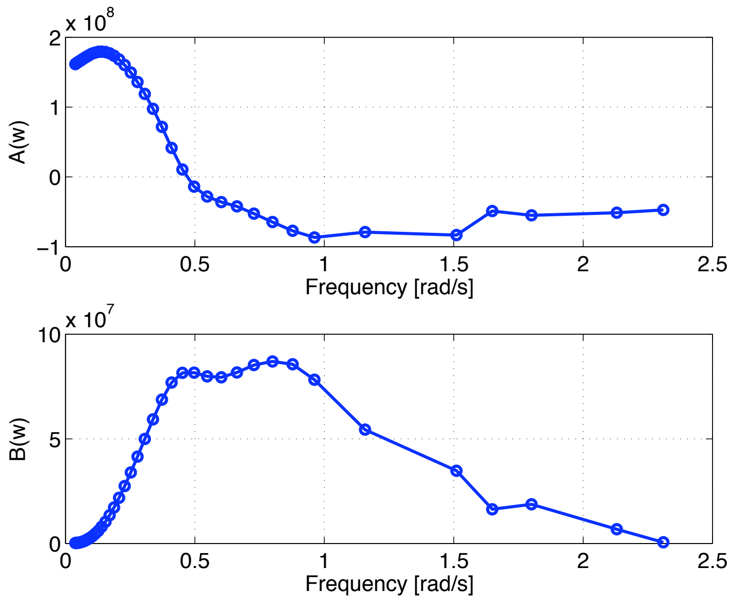

Figure 4: Added mass and damping of a FPSO vessel after eliminating wildpoints. Coupling 5-3 (pitch-heave).

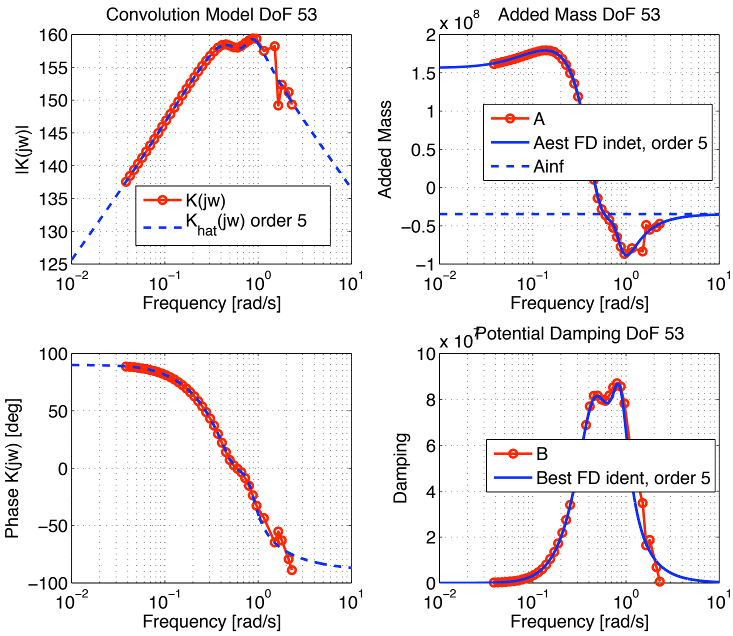

Figure 5: Identification results for coupling 5-3 (pitch-heave) using information of the infinite-frequency added mass. The left-hand plots show the fluid-memory frequency response data and the response of the identified model. The right-hand plots show the added mass and potential damping and the re-constructruction from the identified model.

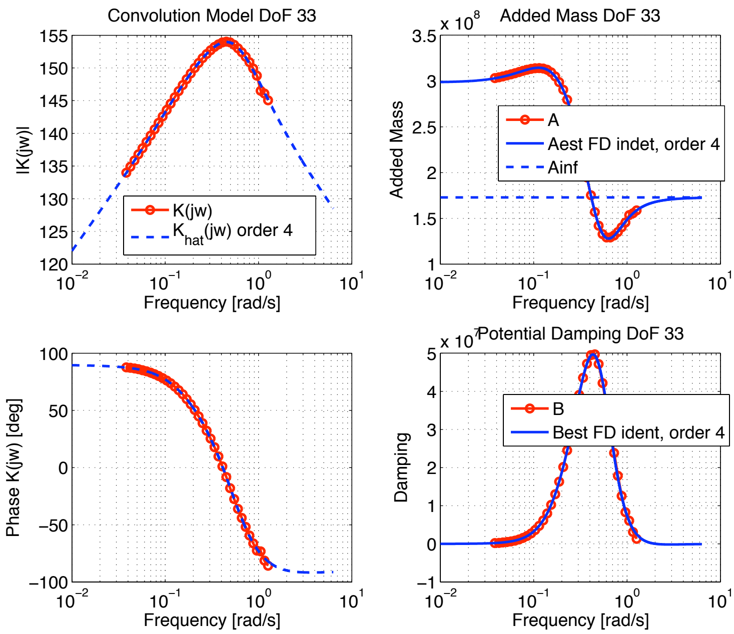

Figure 6: Identification results for coupling 3-3 (pitch-heave) using information of the infinite-frequency added mass. The left-hand plots show the fluid-memory frequency response data and the response of the identified model. The right-hand plots show the added mass and potential damping and the re-constructruction from the identified model.

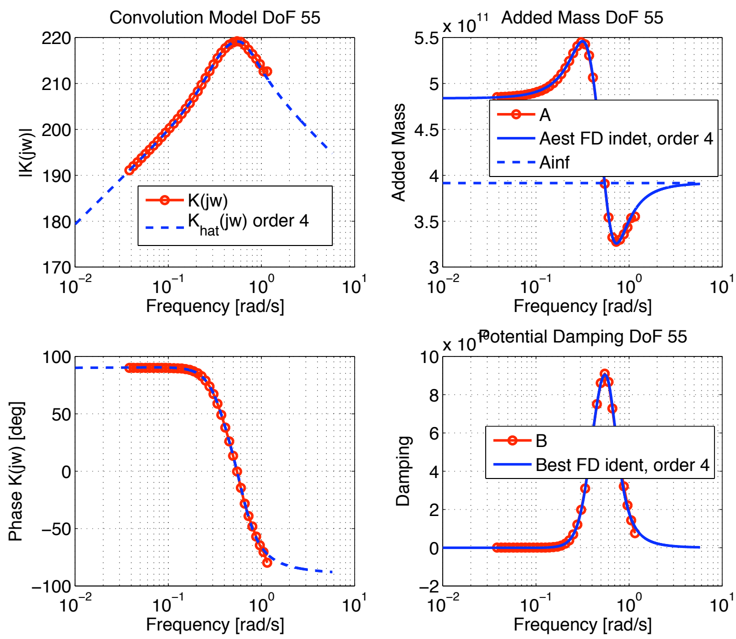

Figure 7: Identification results for coupling 5-5 (pitch-heave) using information of the infinite-frequency added mass. The left-hand plots show the fluid-memory frequency response data and the response of the identified model. The right-hand plots show the added mass and potential damping and the re-constructruction from the identified model.

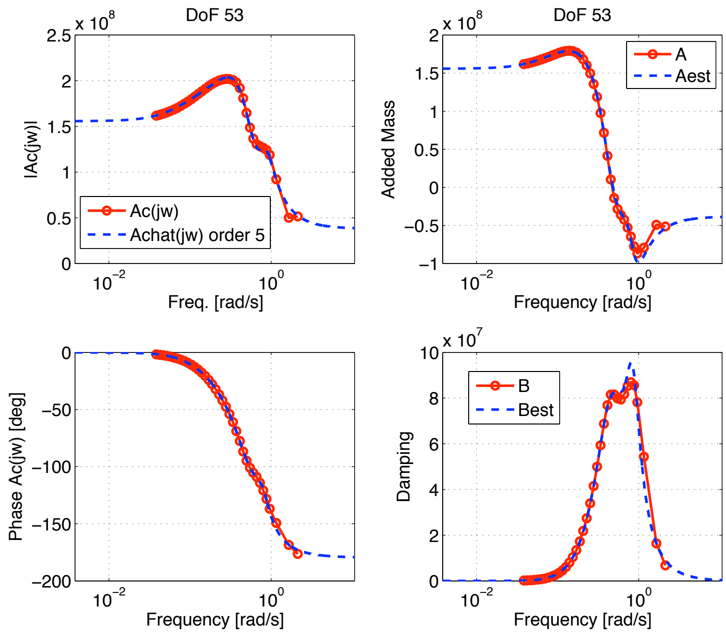

Figure 8: Identification results for coupling 5-3 (pitch-heave) without using information of the infinite-frequency added mass. The left-hand plots show the complex coefficient $\tilde{A}(j\omega)$ data and the response of the identified model. The right-hand plots show the added mass and potential damping and the re-constructruction from the identified model.

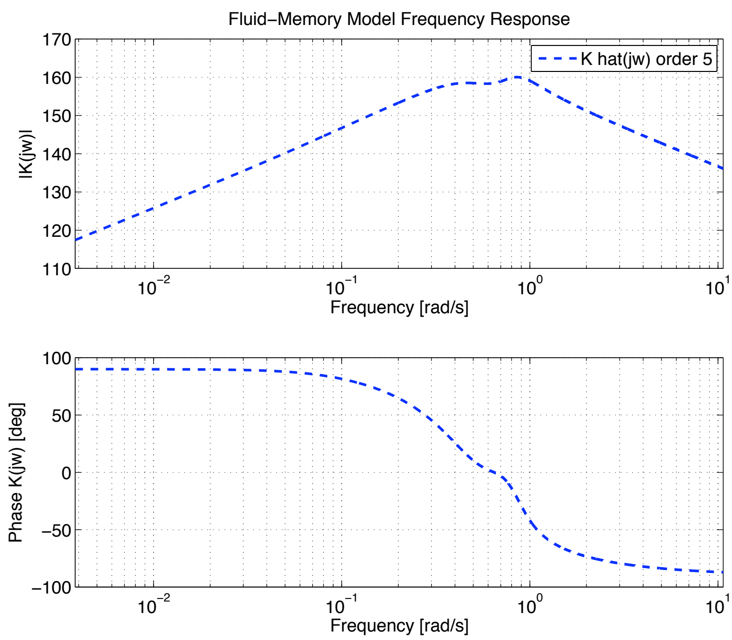

Figure 9: Frequency response of the identified fluid-memory model for the coupling 5-3 (pitch-heave) without using information of the infinite-frequency added mass.

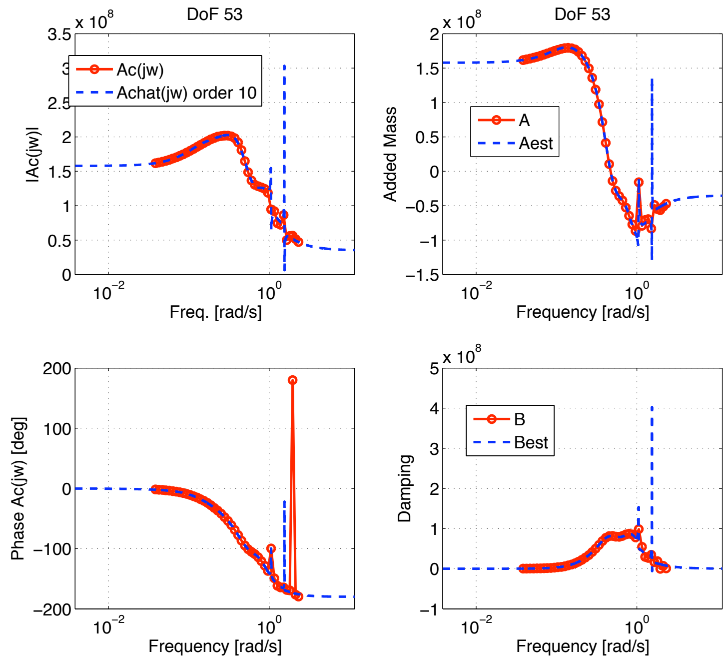

Figure 10: Identification results for coupling 5-3 (pitch-heave) without using information of the infinite-frequency added mass and without eliminating wild points. The left-hand plots show the complex coefficient $\tilde{A}(j\omega)$ data and the response of the identified model. The right-hand plots show the added mass and potential damping and the re-constructruction from the identified model.

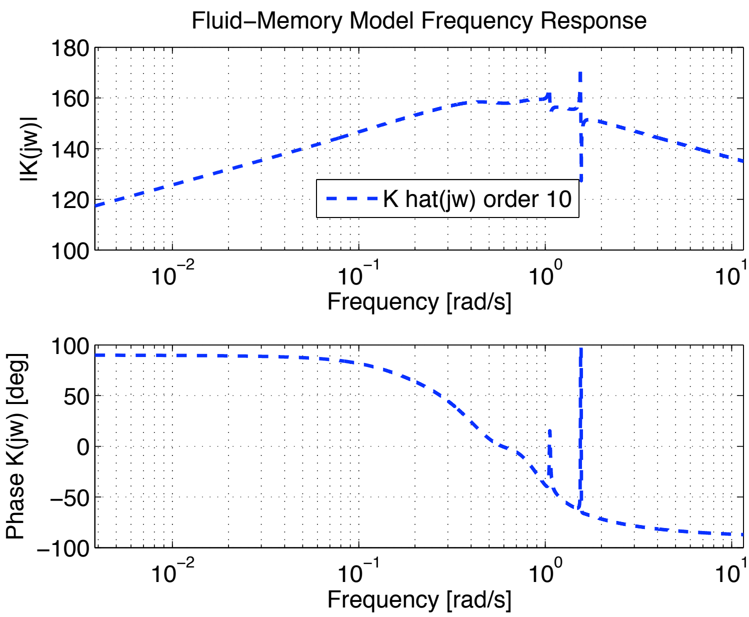

Figure 11: Frequency response of the identified fluid-memory model for the coupling 5-3 (pitch-heave) without using information of the infinite-frequency added mass and without eliminating wild points.